Two Dimensional non-Newtonian MHD Boundary Layer Flow over a Flat Plate with Moving Wedge with Power Law Fluid

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Abstract— In the present paper, we give numerical solution of the Falkner-Skan equation for the study of two-dimensional magnetohydrodynamics steady boundary-layer viscous flow over a wedge in the presence of non-Newtonian power law fluid which is represented by a power-law model. The outer free stream velocity is defined in the form of a power-law manner i.e., it varies as a power of a distance from the leading boundarylayer. The governing nonlinear boundary layer equations have been transformed into a third order nonlinear Falkner-Skan equation through similarity transformations. This equation contains four flow parameters that is the stream-wise pressure gradient (β), the magnetic parameter (M), and boundary stretch parameter (λ), and non-Newtonianpower-law fluid (m). The governing equations (nonlinear partial differential equations) have been converted to an equivalent nonlinear ordinary differential equation along with boundary conditions by means of similarity transformations which is solved using the Keller-box method. A far-field asymptotic solution is also obtained which has revealed oscillatory shapes when the flow has an adverse pressure gradient. The results show that, for the positive pressure gradient parameters, the thickness of the boundary layer becomes thin and the flow is directed entirely towards the wedge surface whereas for negative values the solutions have very different characters. Also it is found that MHD effects on the boundary layer reduce the boundary layer thickness.The results are obtained for velocity profiles and skin friction for various values of physical parameters and are discussed in detail. It is also found that the drag force is reduced for dilatant fluids compared to pseudo-plastic fluids. The Physical significance of the flowparameters are also discussed in detail.

Key words- Boundary-layer equations; Non-Newtonian power-law fluid; Magnetohydrodynamics; Moving wedge; Falkner-Skan equation; Numerical solution; Asymptotic solution;

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1 Introduction

The study of two-dimensional magnetohydro-dynamics (MHD) boundary layer flow of a viscous fluid have attracted large number of researchers during the last decade because of their increasing applications in engineering and technology, such as MHD power generators, MHD flow meters and pumps, polymer industry, spinning of filaments etc.. In industrial applications, when sheets or filaments are subjected to cooling through quiescent fluid, these essentially get stretched, but this cooling of the sheets could be managed by applying the magnetic field, so that we can expect the final products with desired characteristics. Because of such important applications, many investigators have modeled the behavior of a MHD boundary layer flow. Kumars et al. [1] have investigated MHD boundary layer flow of an electrically conducting fluid past a quadratically stretching sheet, and have shown that magnetic field makes the streamlines steeper which results the boundary layer thinner. Joneidi et al. [2] Joneidi undertook the study of heat and mass transfer of a viscous and electrically conducting fluid in the presence of the magnetic

field, and have shown that magnetic field decreases the velocity profiles. Su et. al [3] have used the differential transform method to investigate the MHD Falkner-Skan flow over a permeable plate in the presence of a transverse magnetic field, and have discussed the effects of various physical parameters on the boundary layer flow.

In recent years there has been some interest in the boundary layer flows of non-Newtonian fluids. Many important industrial fluids are non-Newtonian fluids. One particular class of materials of considerable interest is that in which the effective viscosity depends on the rate of shearing on the flow rate. Most particulate slurries (coal in water, sewage sludge and inks, etc.) and multiphase mixtures (oil-water emulsions, gas-liquid dispersions such as froths and foams, butter, etc.) are non-Newtonian fluids as well as melts and solutions of high- molecular weight naturally occurring and synthetic polymers. Other examples of systems displaying a variety of non- Newtonian characteristics include phar-

maceutical formulations (cosmetics and toiletries, paints, synthetic lubricants), biological fluids (blood and saliva, etc.) and food stuffs (jams, jellies, soups, marmalades, etc.). Because such fluids have more complicated equations that relate the shear stresses to the velocityfield that Newtonian fluids have, additional factors must be considered in examining various fluid mechanics and heat transfer phenomena.

One of the main area of interest is the boundary-layer behaviour of a non-Newtonian fluid past a surface in motion relative to either a stationary or moving fluid. This situation represents a different class of boundary-layer problem which has a solution substantially different form that of the boundary-layer flow over a fixed surface and is an important type of flow occurring in a number of material processing applications. Rajagopal et al. [4] looked at the boundary-layer flows of fluids of second grade, and later, Rajagopal et al. [5] studied the Falkner-Skan flows of a homogeneous incompressible fluid of second grade. Andersson and Dandapat [6] were the first who have obtained similarity solutions for the non-Newtonian flow of a power-law fluid past an impermeable stretching surface, while Andersson et al. [7] have extended this problem to the case of a magnetohydrodynamics flow of a power-law fluid over a stretching sheet in the presence of a uniform transverse magnetic field. The boundary-layer flow of a non-Newtonian power-law fluid with injection on a permeable semi-infinite flat plate which moves with a constant velocity in the opposite direction to that of the uniform mainstream has been considered by Akcay and Yijkselen [8]. Kudenatti et al. [9] undertook the study of the exact solution of two-dimensional MHD boundary layer flow over a semi-infinite flat plate. And also Kudenatti et al. [10] have developed similarity solutions of the MHD boundary Layer flow past a constant wedge within porous media. Xu et al. [11] have investigated the boundary layer flow and heat transfer in an incompressible viscous electrically conducting fluid that is caused by impulsive stretching of the surface and used a well-developed homotopy analysis method. They showed that the magnetic parameter reduces the boundary layer thickness but enhances thermal boundary layer thickness.

In this present paper, we have made an attempt to give numerical solution of MHD Falkner-Skan equation of non-Newtonian fluid for general values of β, Hartman number M, power law fluid

m and boundary stretch parameter λ. We find out that the behavior of the boundary-layer flow for various values of the velocity ratio of the plate to different flow parameter and of the power-law index.

2 Formulation of the problem

We consider the two dimensional MHD laminar boundary-layer flow of a viscous and incompressible fluid over a flat plate with wedge which is moving constant velocity $Uw(x)$ in the presence of magnetic field $B(x)$ in a non-Newtonian power-law fluid. The positive x-coordinate is viscous and incompressible measured along the surface of the wedge with the apex as origin, and the positive y-coordinate is measured normal to the x-axis in the outward direction towards the fluid. The fundamental equations for the flow of an incompressible fluid are the conservation of mass, linear momentum.

We express these equations in the absence of body forces as follows:

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\rho\left(\frac{\partial}{\partial t} + \vec{q}.\nabla)\vec{q}\right) = -\nabla p + \nabla \cdot \tau + \vec{J} \times \vec{B} \tag{2}$$

where ρ is the fluid density, p is the pressure, and

$\vec{J} \times \vec{B} = \rho(E + \vec{q}B)B)$ is a Lorentz force. This body force represents the coupling between the magnetic field and the fluid motion which is called Lorentz force. The induced field is assumed to be negligible. This assumption is justified by the fact that the magnetic Reynolds number is very small. This plays a vital role in some engineering problems where the conductivity is not large in the absence of an externally applied field. It has been taken that $E = 0$. Thus the Lorentz force is given by

$$\vec{J} \times \vec{B} = -\sigma B^2 \vec{q} .$$

Since the magnetic drag is a body force on the moving fluid and τ is the deviatoric stress tensor and is defined as

$$\tau = \mu(\dot{q}) \tag{3}$$

where $\dot{q}$ is the second invariant of the strain-rate tensor. The shear rate $\dot{\mathbf{q}}$ is given by

$$\dot{q} = \frac{1}{2}(\dot{\mathbf{q}} : \dot{\mathbf{q}})^{\frac{1}{2}} . \tag{4}$$

with

$$\dot{\mathbf{q}} = (\nabla \vec{u} + \nabla \vec{u}^T) \tag{5}$$

The constitutive viscosity relation μ for the Ostwald-de Waele power-law model is given by

$$\mu = K(\dot{\mathbf{q}})^m \tag{6}$$

where K is the material constant and the index m represents the degree of shear thickening or thinning. We note that the Newtonian viscosity relationship is recovered for $m = 1$. This parameter m is an important index which subdivides the fluids into pseudo-plastic fluids or shear-thickening when $m > 1$ and dilatants or shear-thinning for $m < 1$. Bird et al (1987) can be referred to the through account of the rheological data on m. The hydrodynamics of other values of m shall be discussed later. The velocity vector $\dot{q} = (u, v)$ where u and v are the velocity components in x and y directions respectively, and thus from (4), we have that

$$\dot{q} = \left[\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]^2 + \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right]^{\frac{1}{2}} \tag{7}$$

using (5). We consider the problem of two-dimensional, incompressible and steady state laminar boundary-layer flow over a wedge which moves with velocity $U_{0w}(x)$ in a non-Newtonian power-law fluid. The positive x-coordinate is measured along the surface of the wedge with the apex as origin, and the positive y-coordinate is measured normal to the x-axis in the outward direction towards the fluid. Under these approximations, the governing equations for the steady two-dimensional laminar viscous flow of a non-Newtonian fluid. It is considered that the wedge moves with velocity $U_w(x)$ along or opposite to the mainstream flows $U(x)$. Using the standard boundary-layer approximations and for large R_e we have that $\left| \frac{\partial u}{\partial y} \right| >> \left| \frac{\partial u}{\partial x} \right|$ and $\left| \frac{\partial p}{\partial y} \right| << \left| \frac{\partial p}{\partial x} \right|$. Thus the system (1)-(2) can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \frac{K}{\rho} \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right)^m - \sigma B^2 u \tag{9}$$

Similarly, we get

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{dp}{dy} + \frac{K}{\rho} \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial y} \right)^m - \sigma B^2 v \tag{10}$$

where

$$\dot{q} = \frac{\partial u}{\partial y} \tag{11}$$

To this end, we consider the MHD two-dimensional incompressible flow of the non-Newtonian Ostwald-de Waele power-law fluid over a moving wedge which is moving wedge which is moving with velocity U_{0w} either along the mainstream flow with U_0 or opposite to it. The Cartesian co-ordinate system is adopted to the wedge wall the inviscid main stream velocity U_0 is assumed in the form of power of a distance that is

$$U_0(x) = U_{o\infty} x^{*n} \tag{12}$$

where U_∞ is a non-negative constant and n is a constant related to the pressure gradient defined later in this section. Now, in order to derive boundary layer conditions, the physical quantities and variables specified in (1) and (2) are non-dimensionlized

$$x = \frac{x^*}{L}, \ y = \frac{y^*}{\delta}, \ u = \frac{u^*}{U}, \ v = \frac{v^*}{U}, \ p = \frac{p^*}{p_\infty},$$

L, δ, U and P_∞ are certain reference values. These choices lead to define the Reynolds number for the Ostwald-de Waele power- law fluid as

$$\mathrm{Re} = \frac{\rho \delta^n U^{2-n}}{K}.$$

For a large Re the flow divides in to near-field (boundary- layer region) and far field regions. In the boundary-layer region of thickness of δ, a very large velocity gradient exists. The boundary layer equations can be derived based on the approximations concern the following measurements. Let $U_0(x)$ be the velocity of the mainstream flow along x-direction outside the boundary layer. The key idea involved in making the boundary layer approximation is that the viscosity effects are dominant in the adjacent to the surface. If δ is the thickness of the boundary layer, then $\delta << L$. Hence V is much smaller than U. Also other basic approximation is $\left| \frac{\partial u}{\partial y} \right| >> \left| \frac{\partial u}{\partial x} \right|$. Further, it is also assumed that $\left| \frac{\partial p}{\partial y} \right| << \left| \frac{\partial p}{\partial x} \right|$ in meaning that the pressure p in the boundary layer is a function of x only (to the

approximation). With $\delta << L$ the term $\dfrac{\partial^2 u}{\partial x^2}$ can be neglected in comparison with $\dfrac{\partial^2 u}{\partial y^2}$. Velocity compared to the free stream velocity U_∞ with these assumptions, we have the number of component equations reduce to those in the flow directions. The number of viscous terms in the direction of flow can be reduced to only dominant term. This amounts viscous terms are measured in terms of the boundary-layer thickness. And the inertial terms of the characteristic length L. Thus along with these boundary- layer approximations. Equations (8), (9) and (10) for steady case may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (14)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^m - \frac{\sigma B^2(x)}{\rho} u \qquad (15)$$

$$0 = \frac{\partial p}{\partial y} \qquad (16)$$

where K is called the consistency coefficient and m is non-dimensional, and the dimension of K depends on the value of m. The two-parameter rheological equation (15) is known as the Ostwald-de-Waele model or more commonly, the power-law model. The parameter m is an important index to subdivide fluids into pseudo-plastic fluids $(m < 1)$ and dilatant fluids $(m > 1)$. The extreme cases of the power-law model are $(m = 1)$ for Newtonian behaviour and $(m = 0)$ for plastic or solid behaviour. To determine the pressure distribution, the velocity at the edge of the boundary layer is equal to the mainstream flow $U_0(x)$ and by Bernoullis theorem, the pressure would be constant in the inviscid flow influenced by the applied magnetic field. In order that equations (14) and (15) reduce to similarity form, we assume that the boundary conditions for these equations are of the following form

$$\text{at } y = 0 : u = U_0 w(x), v = 0, \qquad (17)$$

$$\text{as } y \to \infty : u \to U_{0\infty}$$

where $U_0 w(x)$ is the stretching surface velocity which obeys the power-law relation $U_0 w(x) = U_{0\infty} x^m$. In (17), the condition on u

on the surface signifies that the wedge surface is moving, and v_w is constant. The conditions on the velocity at infinity mean that the velocity approaches the mainstream flow far-away from the wedge surface. Thus, the main boundary layer effects are restricted to the immediate neighbourhood of the surface. System (10) and (11) allows reducing both dependent and independent variables to one each by the following similarity transformations. This is further evidenced by the similar velocity profiles existing in the boundary layer for any x in the stream wise direction. The pressure change across the boundary layer is negligible (i.e., constant) and pressure can be treated as function of only flow direction.

Since the pressure is uniform throughout the flow field from the Burnoullis equation, with $u = U_{0\infty}$ outside the boundary layer, we have

$$-\frac{1}{\rho}\frac{dp}{dx} = U_0\frac{dU_0}{dx} \qquad (18)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_0\frac{dU_0}{dx} + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^m$$
$$-\frac{\sigma B^2(x)}{\rho}(u - U_0(x)). \qquad (19)$$

It is clearly observed that the system (14) and (15) with two unknown functions u and v are easily reduced to an equation with one unknown function by defining the stream function $\psi(x, y)$ as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \qquad (20)$$

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = U_0(x)\frac{dU_0(x)}{dx}$$
$$+ \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^m - \frac{\sigma B^2(x)}{\rho}(u - U_0(x)), \qquad (21)$$

with boundary conditions

$$y = 0 : \qquad \frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial y} = 0$$

and as $\quad \dfrac{y}{\partial} \to \infty : \dfrac{\partial \psi}{\partial y} = U_{0\infty}(x)^n \qquad (22)$

The similar solutions of equation (21) can be obtained by using similarity transformation

$$\psi(x, y) = \sqrt{\frac{2KU_\infty x^{1+m^*}}{\rho(n+1)}} f(\eta),$$

$$\eta = \sqrt{\frac{(n+1)\rho U_\infty x^{-1+m^*}}{2K}} y \qquad (23)$$

where

$$m^* = \frac{(3n-1)(m-1)}{m+1}.$$

Substituting (23) in to (21) we get the following ordinary differential equation.

$$\mu_0 f''' + \frac{2}{m+1} ff'' + \frac{\beta}{1+(m-1)\beta}\left(1 - f'^2\right) - M^2\left(f' - 1\right) = 0$$

$$(24)$$

and a new set of boundary conditions,

$$f(0) = 0, \qquad f'(0) = \lambda, \qquad f'(\infty) = 1 \quad (25)$$

where

$$\mu_0(\eta) = m\left|f''(\eta)\right|^{m-1}$$

where primes denote differentiation with respect to η and $\lambda = \dfrac{U_{0w}}{U_{0\infty}}$ is the ratio of the wall velocity to the free stream fluid velocity and $\lambda > 0$ corresponds to the situation when the wall moves in the opposite direction to the free stream (and vice-versa), while $\lambda = 0$ is for a fixed wall. And $\beta = \dfrac{2n}{n+1}$ is the pressure gradient variable parameter. The system (24)-(25) describes the two-dimensional MHD laminar boundary layer flow of a viscous fluid over a moving wedge. Here $\beta > 0$ is the case for a favorable pressure gradient where as $\beta < 0$, it is adverse pressure gradient. For $\beta = 0$ the flow reduces to the well-known Blasius type flow. The parameter $M\left(= \sqrt{\dfrac{2\rho}{\rho U_{0\infty}(m+1)}} B\right)$ is the magnetic (Hartmann number) parameter which is the ratio of electromagnetic field to the viscous force.

3. Numerical solution

Since the analytical solution is usually not possible because of high non-linearity, we resort to solve it numerically using Keller-box method. For this we convert equation into first order system by introducing additional unknown functions as

$$f' = U \qquad (26)$$
$$U' = V \qquad (27)$$

$$BV' + \frac{2}{m+1} fV + \frac{\beta}{1+(m-1)\beta}(1 - U^2) - M^2(U-1) = 0$$

$$(28)$$

where $B = \mu_0 = m\left|V\right|^{m-1}$ and boundary conditions equation (22) becomes

$$f(0) = 0, \qquad U(0) = \lambda \quad \text{and} \quad U(\infty) = 1$$

$$(29)$$

Using the backward finite difference operators for the system (26-28), we get

$$f_j^l - f_{j-1}^l - \frac{\eta_j}{2}\left(U_j^l - U_{j-1}^l\right) = 0$$

$$U_j^l - U_{j-1}^l - \frac{\eta_j}{2}\left(V_j^l - V_{j-1}^l\right) = 0$$

$$BV_j^l - BV_{j-1}^l - \frac{\eta_j}{2(m+1)}\left(f_j^l - f_{j-1}^l\right)\left(V_j^l - V_{j-1}^1\right)$$

$$- \frac{\eta_j \beta}{4(1+(m-1)\beta)}\left(U_j^{l-1} + Uj - 1^{l-1}\right)^2 + \frac{2\eta_j \beta}{1+(m-1)\beta} - M^2 \frac{\eta_j}{2}\left(U_j^l + U_{j-1}^l\right) + 2M^2\eta_j = 0$$

The above system necessarily produces a nonlinear algebraic system of equations for each grid. We linearize the above system by introducing, $f_j^{(l+1)} = f_j^l + \delta f_j^l$, where δf_j^l has to be corrected at each step, we drop the product terms of like $\delta f_j, \delta V_j$ etc and also neglected square terms in δf_j, then, we get

and the boundary conditions are

$$\delta f_0 = \delta U_0 = 0, \quad \delta U_N = 0$$
$$f_0 = 0, U_0 = \lambda, U_N = 1 \qquad (33)$$

$$\delta f_j - \delta f_{j-1} - \frac{\eta_j}{2}\left(\delta U_j + \delta U_{j-1}\right) =$$

$$f_{j-1} - f_j + \frac{\eta_j}{2}\left(U_j - U_{j-1}\right) \qquad (30)$$

$$\delta U_j - \delta U_{j-1} - \frac{\eta_j}{2}\left(\delta V_j + \delta V_{j-1}\right) =$$

$$U_{j-1} - U_j + \frac{\eta_j}{2}\left(V_j - V_{j-1}\right) \qquad (31)$$

$$c_1 \delta V_j + c_2 \delta V_{j-1} + c_3 \delta f + c_4 \delta f_{j-1} + c_5 \delta U_j + c_6 \delta U_{j-1}$$

$$= -\frac{-4d\beta}{1+(m-1)\beta} + BV_{j-1} - BV_j -$$

$$\frac{d}{m+1}\left(f_j V_j - f_{j-1}V_{j-1}\right) +$$

$$\frac{d\beta}{2(1+(m-1)\beta)}\left(U_j + U_{j-1}\right)^2 + \qquad (32)$$

$$M^2 \frac{\eta_j}{2}\left(U_j + U_{j-1}\right)$$

where

$$c_1 = B + \frac{d}{m+1}\left(f_j + f_{j-1}\right),$$

$$c_2 = -B + \frac{d}{m+1}\left(f_j + f_{j-1}\right)$$

$$, c_4 = \frac{d}{m+1}\left(V_j + V_{j-1}\right)$$

$$c_5 = -\frac{d\beta}{1+(m-1)\beta}\left(U_j + U_{j-1}\right) - M^2 d$$

$$c_1\delta V_1 + c_2\delta V_0 + c_3\delta f_1 + c_4\delta f_0 + c_5\delta U_1 + c_6\delta U_0 =$$

$$\frac{-4\delta\beta}{1+(m-1)\beta} + BV_0 - BV_1 - \frac{d}{m+1}\left(f_1 V_1 + f_0 V_0\right) +$$

$$\frac{d\beta}{2(1+(m-1)\beta)}(U_1 + U_0)^2 +$$

$$M^2 d(U_1 + U_0) - 2dM^2$$

Using the above boundary conditions (33) in (30) - (32) we get a block tridiagonal matrix wherein each element is again 3×3 matrix, in the form

$$AD = R \qquad (34)$$

is essentially a linear system.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ -d & 0 & -d \\ c_2 & c_3 & c_1 \end{bmatrix}$$

$$D = \begin{bmatrix} \delta_1, & \delta_2, \dots & \delta_N \end{bmatrix}^T, \delta_j = \begin{bmatrix} f, & U, & V \end{bmatrix}^T,$$

$$R = \begin{bmatrix} R_1, & R_2, & R_N \end{bmatrix}^T, R_j = \begin{bmatrix} R_{11}, & R_{12}, & R_{1N} \end{bmatrix}^T$$

$$\delta_1 = \begin{bmatrix} \delta V_0 \\ \delta f_1 \\ \delta V_1 \end{bmatrix}, \delta_2 = \begin{bmatrix} \delta U_1 \\ \delta f_2 \\ \delta V_2 \end{bmatrix}, \dots \delta_N = \begin{bmatrix} \delta U_{N-1} \\ \delta f_N \\ \delta V_N \end{bmatrix}$$

$$c_6 = -\frac{d\beta}{1+(m-1)\beta}\left(U_j + U_{j-1}\right) - M^2 d$$

and

$$d = \frac{\eta_j}{2}$$

At $j = 1$,
from equations (30 - 32) we get,

$$\delta f_1 - \delta f_0 - d\delta U_1 - d\delta U_0 = f_0 - f_1 + dU_1 - dU_0$$

$$\delta U_1 - \delta U_0 - d\delta V_1 - d\delta V_0 = U_0 - U_1 + dU_1 - dU_0$$

$$A_j = \begin{bmatrix} -d & 1 & 0 \\ -1 & 0 & -d \\ c_6 & c_3 & c_1 \end{bmatrix}$$

$$B_j = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -d \\ 0 & c_4 & c_2 \end{bmatrix}, C_{j-1} = \begin{bmatrix} -d & 0 & 0 \\ 1 & 0 & 0 \\ c_5 & 0 & 0 \end{bmatrix}$$

where j = 2; 3; 4; :::. The tridiagonal structure (37) can be solved using LU decomposition method. The velocity equation for similar for each pressure gradient and magnetohydrodynamic sparameters the Keller-box code also given other required derived quantities such as the velocity profiles. The numerical solution of equation (25) for different parameters M, λ, β, and m has been obtained. Results for the skin frition coefficient, velocity profiles and numerical solutions are reported. It has been shown that multiple solutions are possible when the wedge and the fluid move in the opposite directions, near the region of separation. The drag force is reduced for dilatant fluids compared to pseudo-plastic fluids, these results are affirmed by the asymptotic solution of the governing equations for far-field.

5 Results and Discussion

The similarity solutions of the MHD Falkner-Skan equation for non-Newtonian fluid are obtained for all physical parameters. This equation describes the MHD flow of a viscous fluid over a wedge is moving. The flow is governed by the nonlinear differential equation of order three and is solved by different approaches. The validity and efficiency of the solution method are tested for various parametric values of β, M, m and λ

are compared with the numerical solution of the MHD Falkner-Skan equation. We also investigated the nature of the distribution of velocity in the boundary layer region at which the effects of permeability, magnetic number are taken into account. Numerical values for these parameters are taken which have been extensively used in the previous theoretical studies. In particular, we have taken the range of values for which the solutions are predicted and boundary layer flows are realized. Further, the direct numerical solutions of the MHD Falkner-Skan equation are obtained via finite difference based Keller-box method. This is a standard method for solving nonlinear boundary value problem on a closed interval, in which the Falkner-Skan is converted into an equivalent system of first order equations.The outer boundary condition is taken

at very large value of η, that is, $\eta_{max} \gg 1$. The standard central difference schemes are used for the first order equations, and resulting nonlinear algebraic equations are linearized and solved. Our Keller-box code adapts a variable discretization step size to ensure the desired accuracy in a double precision which was set to 10^{-8} in all our computations. This is because a precise value of $f''(0)$ would be required to compare solution with the numerical ones.

Figure 1 depicts the variation of velocity profiles $f'(\eta)$ as a function of η for different values of magnetic parameter M when the wedge is fixed. There have been simulated using the Keller-box numerical method that is described in section 5. This code starts to predict magnetohydrodynamics fluid (MF) effects on the boundary-layer flow. It is noticed that thickness of the boundary-layer decreases for increasing magnetohydrodynamic fluid. In other words the fluid is attached to the wedge surface. Physically, it is that the magnetohydrodynamic fluid releases more energy to the fluid flow in which the fluid acquires more magnetization. Accordingly, the fluid particles more faster as a result velocity increases. Thus, the boundary-layer thickness naturally decreases. Also, the flow in the magnetohydrodynamic fluid is always stable which is known from the literature that is confirmed by the present studies. Alternatively, when the wedge has same speed $\lambda = 1.5$ From the figure 2 predicts the same features of the boundary-layer for an applied magnetohydrodynamic fluid. In this case also the

boundary-layer thickness decreases for increasing M.

In the figure (1 or 2), the boundary-layer domain is not too large and the Keller-box method converge with 8 iterations exactly when the floerance was set to 10^{-8}. Theseresults are further affirmed by the asymptotic solution (section 4) for various values of M (these have not shown because of the similar nature). Furthermore, figures 1 and 2 are plotted respectively for m = 0.6 (pseudo-plastic fluids) and m = 1.2 (dilatant fluids). It is noticed that whether for shear-thinning boundary-layer (m < 1) or shearthickening boundary-layer (m > 1), the magnetohydrodynamic fluids the same effects on the study two-dimensional boundary-layer flow.

Alternatively, the effects of Ostawald-de-waele non-Newtonian power-law fluid on the boundary-layer flow are studied. Figure 3 explored this effect for various values ofm and two set of _ and keeping others constant. The effect of the power-law fluidis clearly distinguished from that of the Newtonian case (m = 1). The boundarylayer thickness for the for the shear thinning fluids (m < 1) is quite smaller compared to their counter-parts (m > 1). In otherwords, the viscosity, effects are predominanton the boundary-layer of the shear thinning fluids compared to shear thickening fluids (m > 1). The magnetic field has the same effects on the flow irrespective of $m \neq 1$ as discussed in figure 1 and 2. It is also noticed that when λ = 0, all the profiles approachtheir end condition from the below (figure 3a) and profiles in the Figure 3b approach from the above in the speed. In the case of figure 3b results, the wedge speed is 1.5 times grater than that of the mainstream velocity, hence, the profiles approach from above Dabrowiski and Denier (2004) have shown for λ = 0 that irrespective values of m, the profile approach their-end condition algebraic manner i.e. $f'(\eta) = 1 + A\eta^{\frac{m+1}{m-1}}$, where A some constant depending on m and $m \neq 1$. From the computational point of view, the Keller-box code converges to the desired solutions (mainly

$f'(\eta)$ and the wall shear-stress $f''(0)$ in

8^3 iterations for all values of m and λ tested. Add something on computation.

We now move on to discuss other important features of boundary-layer flow in terms the viscosity profiles. These are given by (figures 5-8). Our Keller-box code simultaneously simulates these viscosity profiles along with the velocity profiles

for all the system parameters. The case for *m = 1* (Newtonian fluid) demarcates the viscosity profiles from those of $m \neq 1$. The viscosity shapes $\mu_0(\eta)$ as a function of η for varies values of the shear-thinning fluids *(m < 1)* are plotted in figure 4. As in the case of the Newtonian fluid where the viscosity becomes constant (see figures 5-8), the viscosity solutions grow unboundedly within the confines of the boundary-layer as η increases from 0. For smaller values of *m* say (0.6) the increase of the solutions is more than exponential ans has m increases to 1, this becomes linear. We note that these viscosity profiles further increase for increasing either the pressure gradient β or the magnetic field parameter M as is seen clearly in figures 5. This shows that $\mu_0(\eta) \rightarrow \infty$ as $\eta \rightarrow \infty$ all parameters when *m < 1*.

Furthermore, figures 7 and 8 predicts the viscosity shapes μ_0 for he shear-thickening fluids *(m > 1)* which are quite different from those of figure 4 compared to figures 5

and 6. Since *m > 1*, $\mu_0 \rightarrow 0$ as $\eta \rightarrow \infty$ for any choice of other parameters. The shear flow $f''(\eta)$ tends to zero as $\eta \rightarrow \infty$, the viscosity μ_0 given by (figures 7-8) also tends to zero. In otherwords, the viscosity solutions become zero near the edge of the boundary-layer. There is an exponential decrease to zero as $\eta \rightarrow \infty$, and this rapidness of the decrease is rather more for largevalues of m say (1.6)
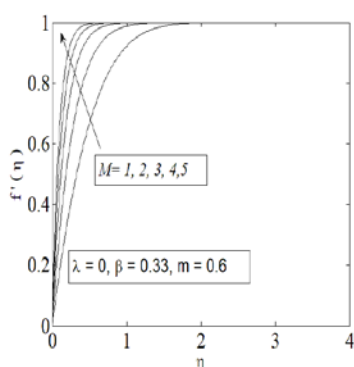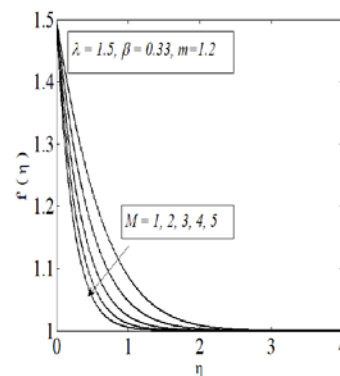

Fig.2. Variation of velocity profiles $f'(\eta)$ with η for $M = 1,2,3,4,5$, $\lambda = 1.5$, $\beta = 0.33$ and $m = 1.2$.

Fig. 1. Variation of velocity profiles $f'(\eta)$ with η for $M = 1,2,3,4,5$, $\lambda = 0$, $\beta = 0.33$ and $m = 0.6$.
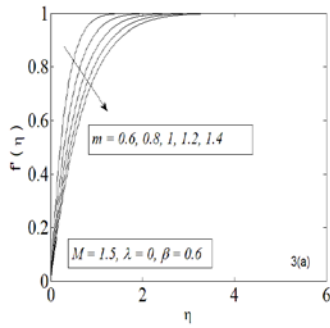
Fig.3(a). Variation of velocity profiles $f'(\eta)$ with η for $m = 0.6, 0.8, 1.0, 1.2, 1.4$, $\lambda = 0$, $\beta = 0.6$ and $M = 1.5$.
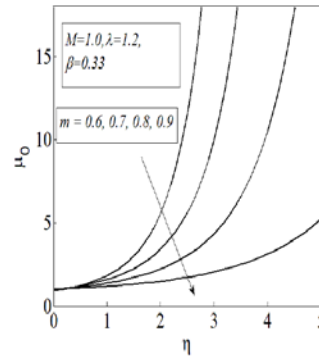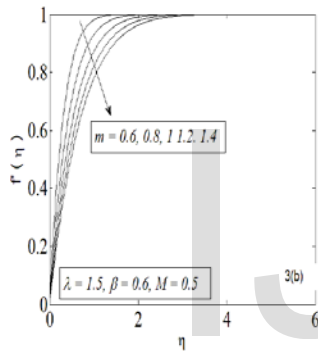

Fig.4. Variation of velocity profiles μ_0 with η for $m = 0.6, 0.7, 0.8, 0.9$, $\lambda = 1.2$, $\beta = 0.33$ and $M = 1.0$.

Fig.3(b). Variation of velocity profiles $f'(\eta)$ with η for $m = 0.6, 0.8, 1.0, 1.2, 1.4$, $\lambda = 1.5$, $\beta = 0.6$ and $M = 0.5$.
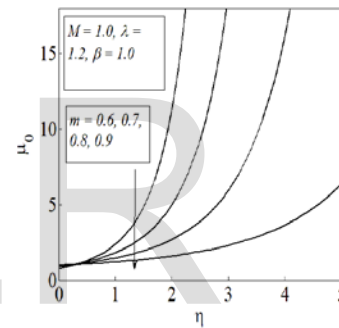

Fig.5. Variation of velocity profiles μ_0 with η for $m = 0.6, 0.7, 0.8, 0.9$, $\lambda = 1.2$, $\beta = 1.0$ and $M = 1.0$.
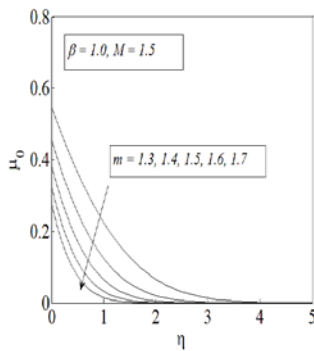
Fig.6. Variation of velocity profiles μ_0 with η for
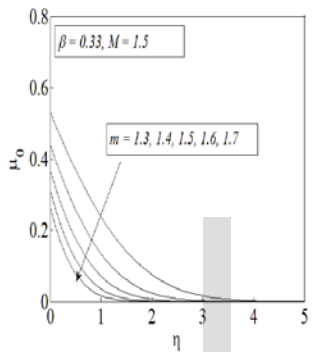$m = 1.3, 1.4, 1.5, 16, 1.7$, $\beta = 1.0$ and $M = 1.5$.

Fig.7. Variation of velocity profiles μ_0 with η for
$m = 1.3, 1.4, 1.5, 16, 1.7$, $\beta = 0.33$ and $M = 1.5$.

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